Title: The geometry of a counting formula for deformations of the braid arrangement

Abstract:
We consider real hyperplane arrangements whose hyperplanes are of the form $\{x_i - x_j = s\}$ for some integer s, which we call *deformations of the braid arrangement*. In 2018, Bernardi gave a counting formula for the number of regions of any deformation of the braid arrangement $\mathcal{A}$ as a signed sum over some decorated trees. He further showed that each of these decorated trees can be associated to a region R of the arrangement $\mathcal{A}$, and hence we can consider the contribution of each region to the signed sum. Bernardi also implicitly showed that for *transitive* arrangements, the contribution of any region of the arrangement is 1. We remove the transitivity condition, showing that for *any* deformation of the braid arrangement the contribution of a region to the signed sum is 1. This provides an alternative proof of the original counting formula, and sheds light on the geometry underlying the formula. We further use this new geometric understanding to better understand the contribution of a tree. This is based on joint work with Aaron Lin.

Arxiv link: https://arxiv.org/abs/2603.24885

Notes:

I would like to have this considered for the Best Paper Presentation Awards.

I have already registered and paid for the conference with the email ngoregaokar@brandeis.edu.